

# Comment on “Drainage of a Thin Liquid Film between Hydrophobic Spheres: Boundary Curvature Effects”

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We clarify the basic faults in Fang, A.; Mi, Y. Drainage of a Thin Liquid Film between Hydrophobic Spheres: Boundary Curvature Effects. *Langmuir* 2014, 30, 83-89, which led to unphysical conclusions.

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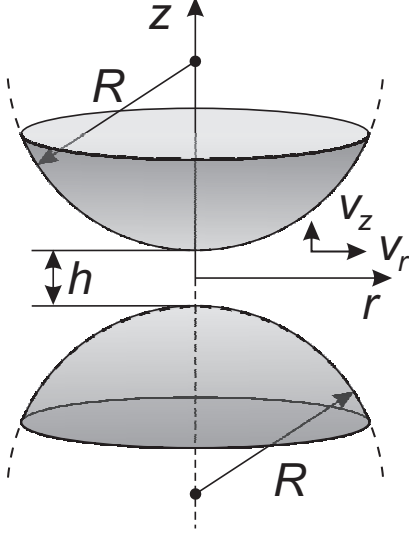


FIG. 1: Definition sketch for two approaching (with the velocity  $V$ ) spheres of radius  $R$  separated by the gap  $h = \varepsilon R$ , where  $\varepsilon \ll 1$ . Cylindrical coordinates  $(r, z)$  are used with origin in the center of the gap, so that the equations of the sphere surfaces become  $\pm H(r) \simeq \pm \frac{h}{2} \pm \frac{r^2}{2R}$ . The radial and normal velocity components are  $v_r$  and  $v_z$ .

In their paper [1] Fang and Mi declare that by including the *boundary curvature effect* into the hydrodynamic slip boundary conditions, they reformulate and improve the Vinogradova theory of a thin film drainage between hydrophobic surfaces. [2] As a side note, we remark that results [2] are general, being applicable, besides hydrophobic surfaces in water, [3, 4] to a description of a drainage of confined polymers, [5, 6] gases [7, 8], or any other systems characterized by a partial slip length,  $b$ .

The solution by Fang and Mi significantly underestimates the force and does not recover the classical Taylor (Reynolds) result [9] expected at large distances,  $h \gg b$ . Besides that, a curvature contribution to the slip length [10], underlying the boundary conditions [1], should obviously become important only for spheres of a small radius, but not with numerical parameters used by Fang and Mi.

In the present Comment, we clarify the mistakes by Fang and Mi, [1] and point out that the correction to the Vinogradova theory due to an extra curvature term cannot be derived within the lubrication approximation.

There are at least three basic faults in Ref. [1] which lead to erroneous conclusions. To highlight them we focus on the case of two identical spheres studied analytically by Fang and Mi (see Fig. 1).

*First*, the lubrication approximation, Eqs. (12) and (13) by Fang and Mi, implies that  $\varepsilon \ll 1$ , so that  $v_r = O(\varepsilon^{-1/2}V)$  and  $r = O(\varepsilon^{1/2}R)$ . In such a case in the set of Eqs.(10) and (11) left ones are fully identical to formulated by Vinogradova, [2] and include the boundary curvature term,  $\frac{rv_r}{R} = O(V)$ . Additional terms due to curvature,  $\frac{rV}{R} = O(\varepsilon^{1/2}V)$ , included by Fang and Mi into right equations of (10) and (11), should be neglected in the leading-order solution being out-of-order. These could only be included if terms of the same order are kept in Eqs. (5), (6), (8) by Fang and Mi. To construct then the second-order corrections to pressure and velocity fields it is necessary to match the inner solution in the gap with the outer solution at distances  $r = O(R)$ . Such a problem is certainly beyond the scope of the lubrication approximation.

*Second*, integrating the continuity equation, Eq.(7) by Fang and Mi, over  $z$  should be made as follows [11]:

$$\begin{aligned} \int_{-H(r)}^{H(r)} \left[ \frac{\partial v_z}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} (rv_r) \right] dz &= v_z(H) - v_z(-H) \\ &+ \frac{1}{r} \frac{d}{dr} \int_{-H(r)}^{H(r)} rv_r dz - \frac{dH}{dr} (v_r(H) + v_r(-H)) = 0. \end{aligned}$$

By substituting here  $v_z(H)$  and  $v_z(-H)$  from their Eqs.(10) and (11) we then obtain

$$\frac{r}{R} [v_r(H) + v_r(-H)] - V + \frac{1}{r} \frac{d}{dr} \left( \int_{-H(r)}^{H(r)} rv_r dz \right) - \frac{dH}{dr} [v_r(H) + v_r(-H)] = 0, \quad (1)$$

and conclude that Fang and Mi missed the last term in our Eq.(1). Indeed, the first and last terms are obviously canceled out since  $\frac{dH}{dr} \simeq \frac{r}{R}$ , and we get

$$-V + \frac{1}{r} \frac{d}{dr} \left( \int_{-H(r)}^{H(r)} r v_r dz \right) = -V + \frac{1}{r} \frac{d}{dr} \left( -\frac{H^3}{3\mu} \frac{dp}{dr} + 2C_2 H \right) = 0. \quad (2)$$

Thus, constant  $C_1$  in the velocity profile, Eq.(14) by Fang and Mi, which they claim to represent a “curvature-induced renormalization of the slip length”, does not contribute to the integral of the continuity equation.

*Third*, Eq.(15) derived by Fang and Mi gives a non-zero value for constant  $C_1$ , which implies that their velocity profile is asymmetric. Since the problem for two equal spheres is symmetric,  $v_r$  is necessarily an even function of  $z$ , which requires  $C_1 = 0$ . This error is likely caused by a wrong sign for one of the boundary curvature terms,  $\frac{rV}{R}$ , in their Eqs. (10) and (11).

In summary, although the extension of the Vinogradova theory [2] to the case of highly curved surfaces would be very timely, the theory by Fang and Mi [1] fails in trying this.

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